LP rounding

$$P_j := \{ i : x_i \in C_j \} \quad N_j := \{ i : \neg x_i \in C_j \}$$

$$\Pr[C_j \text{ is sat}] = 1 - \prod_{i \in P_j} (1 - y_i^*) \cdot \prod_{i \in N_j} y_i^* \overset{(I)}{\geq} 1 - \left(\frac{\sum_{i \in P_j} (1 - y_i^*) + \sum_{i \in N_j} y_i^*}{|C_j|} \right)^{|C_j|}$$

$$\overset{(II)}{\geq} 1 - \left(1 - \frac{z_j^*}{|C_j|}\right)^{|C_j|} \overset{(III)}{\geq} \Bigg(1 - \underbrace{\left(1 - \frac{1}{|C_j|}\right)^{|C_j|}}_{\leq \exp(-1)}\Bigg) z_j^*$$

$$\geq \left(1 - \frac{1}{e}\right) z_j^*$$

$$\Rightarrow \mathbb{E}[ALG] \geq \left(1 - \frac{1}{e}\right) \cdot \sum_{j=1}^{m} w_j z_j^* \geq \left(1 - \frac{1}{e}\right) \cdot OPT \quad \square$$

Geometric vs. Arithmetic Mean:

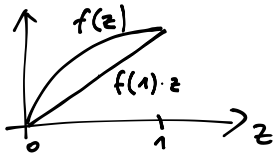
(I) $\left(\prod_{i=1}^{k} a_i \right)^{\frac{1}{k}} \leq \frac{1}{k} \sum_{i=1}^{k} a_i$ for any $a_1, \dots, a_k \in \mathbb{R}_+$

(II) $$\sum_{i \in P_j} (1 - y_i^*) + \sum_{i \in N_j} y_i^* = |C_j| - |P_j| - |N_j| - \left(\sum_{i \in P_j} (y_i^* - 1) + \sum_{i \in N_j} -y_i^* \right)$$
$$= |C_j| - \left(\sum_{i \in P_j} y_i^* + \sum_{i \in N_j} (1 - y_i^*) \right) \leq |C_j| - z_j^*$$

(III) $f(z) = 1 - \left(1 - \frac{z}{k}\right)^k$ is concave for $z \in [0, 1]$.

$$f(0) = 0 \qquad f(1) = 1 - \left(1 - \frac{1}{k}\right)^k$$

$$\Rightarrow f(z) \geq \left(1 - \left(1 - \frac{1}{k}\right)^k\right) \cdot z$$



Non-linear Randomized Rounding

$$\Pr[C_j \text{ not sat}] = \prod_{i \in P_j} (1 - f(y_i^*)) \prod_{i \in N_j} f(y_i^*) \leq \prod_{i \in P_j} 4^{-y_i^*} \prod_{i \in N_j} 4^{y_i^* - 1}$$

$$= 4^{-\left(\sum_{i \in P_j} y_i^* + \sum_{i \in N_j} 1 - y_i^*\right)} \leq 4^{-z_j^*}$$

$$\Pr[C_j \text{ sat}] \geq 1 - 4^{-z_j^*} \overset{\text{as in (III)}}{\geq} (1 - 4^{-1}) z_j^* = \frac{3}{4} z_j^*$$

$$\mathbb{E}[ALG] \geq \sum_{j=1}^{m} w_j \Pr[C_j \text{ sat}] \geq \frac{3}{4} \sum_{j=1}^{m} w_j z_j^* \geq \frac{3}{4} OPT \quad \square$$

not discussed in lecture